HEAT TRANSFER IN TURBULENT FLOW AT VERY LOW PRANDTL NUMBERS

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An analysis is performed of available experimental data on heat transfer in the region of very low Prandtl numbers $0{\cdot}005 \leqq Pr \leqq 3$. None of the verified models on heat transfer mechanism agrees with experimental data in the whole studied region. With regard to a small number of experimental data it is not possible to make a final decision whether the disagreement is due to inaccuracies in experiments or on the side of theory.

The earlier published[1] theoretical model on the mechanism of heat or mass transfer in turbulent liquid flow has been verified experimentally, by data published in literature and by our own experimental data[2-4]. So has been verified its suitability roughly in the region $0{\cdot}03 < Sc(Pr) < 3\,000$ and in the region $10^3 \leqq Sc \leqq 10^6$. In the region of very small Prandtl numbers $3 \cdot 10^{-3} \leqq Pr \leqq 10^{-2}$ which are of significance for heat transfer with liquid metals, the model has been verified for only two close values of Pr ($Pr = 0{\cdot}027$ and $0{\cdot}0243$). But this verification seems insufficient as with liquid metals used in nuclear energetics values of Pr of the order of magnitude 10^{-3} are frequently encountered. Therefore an analysis has been made here of the available experimental data on heat transfer at turbulent flow in the region of approx. $0{\cdot}005 \leqq Pr \leqq 3$ and they have been compared with values predicted according to the theoretical model[1] and other theoretical models. For our purposes, data obtained on the circular pipe have been used where the data on heat transfer at boiling point were eliminated.

THEORETICAL

The verified theoretical dependence for heat transfer in a smooth pipe has a form

$$k^+ = \frac{F(Pr)}{Pr\left[F(Pr)\,\delta^+ + \lambda^+ + 0{\cdot}3153 Re^{0.1875}\Phi(Pr)\right]}. \qquad (1)$$

The relation (1) for $Pr \leqq 0{\cdot}03$ can be arranged with accuracy better than 1% into the

form

$$k^+ = 1/0{\cdot}3152(2 - Pr)\, Re^{0{\cdot}1875}, \qquad (2)$$

where similarly as in Eq. (*1*) the familiar Blausius relation (*3*) for the smooth pipe has been used for expressing the friction factor f. From Eq. (*2*) results that in the region of very low Pr numbers the verified model assumes negligibly small dependence of k^+ on Prandtl number.

RESULTS

Experimental Data

It is possible to find in literature a considerable number of experimental studies concerning the heat transfer with liquid metals. But the majority cannot be used for verification of theoretical models since under equal experimental conditions the results of individual authors are different, data have a large scatter and in papers are not given the physico-chemical constants. Summary of the already published experimental data given by Lubarski and Kaufman[5] is demonstrated in Fig. 1. It can be seen from this Fig. that the scatter of data is considerable, inspite of the fact that Lubarski and Kaufman[5] tried to unify the data by their recalculation to the same experimental conditions and for identical values of physicochemical constants. The experimental data are mostly published in the form $Nu = f(Pe)$. This form of dependence is disadvantageous as it does not enable to differentiate among the data for various values of Pr and similarly it is not capable to distinguish the data obtained at laminar or turbulent flow regimes. We have attempted to express in this study on basis of the verified dependence of dimensionless heat transfer coefficient k^+ on Re for individual Pr numbers in log coordinates. It is very difficult to obtain the experimental data from publications for the dependence of this type, as the authors do not publish original data and so it is not possible to recalculate the form $Nu = f(Pe)$ to the form $k^+ = f(Re, Pr)$.

In Table I is summarised the survey of used experimental studies. In this selection we have attempted to select such sets of data where for the given value of Pr it is possible to obtain as wide dependence as possible of k^+ on Re so that the data have an acceptable scatter. The original studies with dependences of the type $Nu = f(Pe)$ were included into Table I only in the case when the given range of Prandtl numbers was narrow and recalculation of data at the use of the mean Pr value could have been performed with sufficient accuracy. Otherwise the experimental data were neither arranged nor corrected. For the friction factor relations (*3*) and (*4*) were used

$$f = 0{\cdot}0791 Re^{-0{\cdot}25} \quad 2\,300 < Re \leqq 10^5 \qquad (3)$$

$$f = 0{\cdot}046Re^{-0.20} \qquad Re > 10^5 \qquad (4)$$

Certain studies from heat transfer into gases and liquids were also included into Table I. The reason was the possibility of comparison of regions of liquid metals and the subsequent regions of Prandtl numbers of gases and liquids. Experimental data from publications in Table I are given in Fig. 2 from which is obvious that for the constant value of Prandtl number the dependence of k^+ on Re is approximately linear. The slope of this linear dependence is a function of Prandtl number, and so with increasing Pr

TABLE I
Source of experimental data

Ref.	Liquid	Pr	$\overline{Pr}$	Conditions on wall Q_w = const. or T_w = const.	Data obtained
6	Pb–Bi		0·029	Q_w	read off graph in study[19]
7	Hg	0·014—0·018		T_w	calc. from exp. data
8	Na	0·0057—0·0075	0·0066	Q_w	read off graph
9	Al	0·0139—0·0143	0·014	Q_w	calc. from exp. data
10	Na–K	0·0224—0·028		Q_w	calc. from exp. data
11	Na	0·006 —0·007	0·0065	Q_w	read off graph in study[21]
12	Hg–Mg		0·005	Q_w	read off graph
13	Na–K	0·0203—0·0245		T_w	calc. from exp. data
14	H_2–N_2	0·46 —0·475		T_w	read off graph in study[20]
15	air		0·7	T_w	read off graph in study[19]
16	air		0·7	T_w	read off graph in study[19]
17	air		0·7	T_w	read off graph in study[19]
18	H_2O		3	T_w	read off graph in study[19]

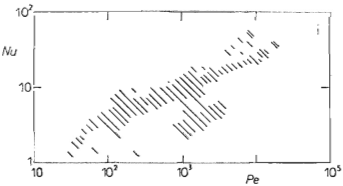

FIG. 1
Experimental data on heat transfer in the region of "liquid metals"

number the slope in absolute value decreases. For heat transfer into gases a very slight dependence of k^+ on Re is obtained and for heat transfer into water k^+ is independent of Re. But this fact could be affected to a certain extent by the magnitude of the used set of experimental data. Other suitable experimental data were not available.

Analysis of Published Results

As can be seen from Fig. 2, majority of experimental data on the dependence of k^+ on Re in a sufficiently wide range of Reynolds numbers exist for values of $Pr = 0{\cdot}005$, $0{\cdot}025$, $0{\cdot}7$ and 3. For these Pr numbers is made in Fig. 3 the graphical comparison of theoretical and experimental results. Experimental data from Table I are compared both with the dependence according to the considered model (Eq. (*1*)) and with other theoretical and empirical relations.

The most familiar and most frequently used theories of mass (heat) transfer mechanism are the eddy diffusivity (thermal conductivity) theory. As is typical for this theory, at derivation of relations for k^+ both values of the molecular Prandtl number Pr and also of the turbulent Prandtl number Pr_t are met. For the first approximation Pr_t has been chosen equal to one as is used *e.g.* in correlations by Martinelli[22] and Deissler[23]. It can be seen from Fig. 3 that the Martinelli theory has the best agreement for $Pr = 0{\cdot}7$. For other values of Pr the shape of theoretical dependence does not correspond to that of experimental data. Moreover the calculation according to Martinelli is rather complex. In our case the values of k^+ were obtained by interpolation of the tabulated values[33]. As concerns the correlation by Deissler, it is in a very good agreement for $Pr = 0{\cdot}005$ but on the other hand it completely fails

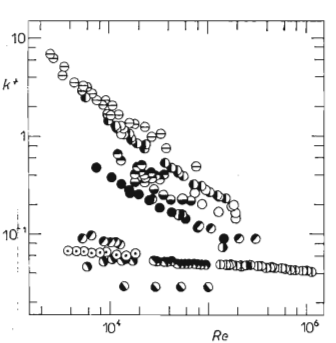

FIG. 2

Experimental data on heat transfer in the region of low Prandtl numbers. ● ref.[6], ◑ ref.[7], ◐ ref.[8], ◓ ref.[9], ◒ ref.[10], ⊖ ref.[11], ○ ref.[12], ◴ ref.[13], ◪ ref.[14], ◔ ref.[15], ⊙ ref.[16], ⦶ ref.[17], ◔ ref.[18]

for $Pr = 0{\cdot}7$ and 3. It is thus obvious that the assumption $Pr_t = 1$ is a too great simplification. The theory of eddy diffusivity is thus reduced to finding the most suitable relation for Pr_t. The most advanced approach in this respect was made by Rohsenow and Cohen[24] who had already derived a special relation for Pr_t. For $Pr > 0{\cdot}1$ they set[25] $Pr_t = 1$ and their theory is taking the form of the relation by Martinelli[22]. From Fig. 3 can be seen that the dependence $k^+ = f(Re, Pr)$ calculated[25] on basis of the relation by Rohsenow and Cohen is passing through the central part of experimental data at $Pr = 0{\cdot}005$, however for $Pr = 0{\cdot}025$ the agreement is not so good. Theory by Rohsenow and Cohen[24] is considered by White[26] as the best available one for Pr_t at present. Jischa and Rieke[27] demonstrate that the experimentally determined dependences of Pr_t on Pr or Re "are different and have a wide spread." They also state that the "experimental knowledge in this field is rather poor and that it is risky to draw conclusions from unsecured experimental data ..."

The best fit is obtained by empirical correlations, usually only in a narrow range of variables. Among them belongs the correlation by Lubarski and Kaufman[28]. This correlation has been formed on basis of experimental data with $Pr = 0{\cdot}02$ to $0{\cdot}03$. Therefore, as is also obvious from Fig. 3, for $Pr = 0{\cdot}025$ a very good fit has been obtained. For $Pr = 0{\cdot}005$ is the agreement considerably worse and for $Pr \geqq 0{\cdot}7$ the correlation fails.

The verified theoretical[1] relation is also plotted in Fig. 3. As we can see, the originally good agreement for $Pr = 3$ and $0{\cdot}7$ disappears with decreasing Pr so that for $Pr = 0{\cdot}005$ only a qualitative information is obtained. But when the wide range of validity of this relation is taken into consideration, even in the region of $Pr \gg 3$ the agreement is quite good. The reason of greater disagreement can be found in hydro-

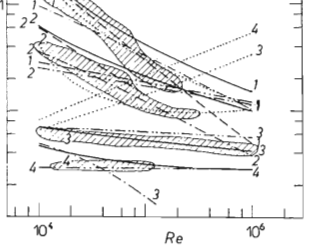

FIG. 3

Comparison of experimental data with theory –·–·– Kolář[1], ——— Martinelli[22], ····· Deissler[23], Rohsenow and Choi[25], –·–·– Lubarski and Kaufman[28], /////// experimental data. *1* $Pr = 0{\cdot}005$, *2* $Pr = 0{\cdot}025$, *3* $Pr = 0{\cdot}7$, *4* $Pr = 3$

dynamics in vicinity of the phase interface differing from that assumed in the model which is based on results of hydrodynamic studies performed only in usual liquids and gases. In liquid metals the hydrodynamics may differ due to high density. Another factor is that majority of experimental data for liquid metals was obtained under condition of constant heat flux on the wall and not at constant temperature as was assumed in the theoretical model[1]. For consideration of possible effect of conditions on interface, experimental data are not available.

DISCUSSION

As concerns the experimental data from the region of heat transfer into liquid metals it is necessary to emphasize how difficult it is to obtain them, which is even more difficult when higher accuracy is required. The precision is influenced by rather high temperature necessary for keeping the metal in liquid state (with the exception of Hg) and by related problems with thermal insulation of the experimental section. With the change of temperature resulting from the heat transfer is also related the change in physico-chemical properties over the cross-sectional area as well as along the experimental section[29,30]. This problem is usually not met at mass transfer as the physico-chemical properties of compounds depend only very little on concentration. It is also interesting that values of physico-chemical data can be affected by impurities present in the liquid either by the surface active compounds, impurities of other metals or content of absorbed gases[31,32]. With impurities on the interface is also related the question of the contact angle and its effect on the resistance toward heat transfer as liquid metals usually do not wet the wall of the experimental section.

Another problem related with the change in temperature is the choice of reference temperature at which the values of dimensionless criteria are determined. This question was studied *e.g.* by Sleicher and Rouse[30]. On basis of available experimental data they found that within the range $0{\cdot}1 < Pr < 10^5$ and $10^4 < Re < 10^6$ the best fit of data and theory was obtained using bulk temperature at Nusselt number, film temperature at Reynolds number and wall temperature at Prandtl number. Choice of these temperatures may differ from case to case according to the considered mechanism of heat transfer.

Experimental data are the only source of information for verification of the theory and therefore they are exposed to considerable requirements. To eliminate eventual shortcomings in validity of the theoretical model, it is necessary to have for verification at the disposal several sets of experimental data by various authors for identical experimental conditions. From this point of view the amount of experimental data presented in this paper is small and not sufficient for final conclusions.

From the made conclusions it can be seen that study of heat transfer into liquid metals is a very complex question and it can happen that with the improved experi-

mental technique in several years neither the data presented here will be considered accurate.

LIST OF SYMBOLS

f friction factor
f function
F function in Eq. (*1*)
k^+ dimensionless fully developed heat transfer coefficient
Nu Nusselt number
Pe Peclet number
Pr Prandtl number
Pr_t turbulent Prandtl number
Re Reynolds number
δ^+ dimensionless thickness of laminar layer
λ^+ dimensionless thickness of transition layer
Φ function in Eq. (*1*)

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